



Getting it Right!



Unit Topic:	Navigation
Grade Level:	7 th grade (with suggestions to scale for grades 6 to 8)
Lesson No.	4 of 10
Lesson Subject(s):	Calculation, approximation, geometry, trigonometry
Key Words:	Accuracy, precision, calculation, approximation, geometry, trigonometry

Lesson Abstract —

In this lesson, students will investigate *error*. As shown in earlier activities, without understanding how errors can affect your position you cannot navigate well. Introducing accuracy and precision will develop these concepts further. The students will also learn about how computers can help in navigation. Often, the calculations needed to navigate accurately are time consuming and complex. By using the power of computers to do calculations and repetitive tasks, one can quickly see how changing parameters like angles and distances and introducing errors will affect their overall result. These concepts will be reviewed in the following activities:

- Close Enough – students will learn about the difference between accuracy and precision. Using a computer, they will see how much accuracy is needed for various activities. If going to the moon, one needs to have a very accurate flight path or the moon might be missed completely!
- Computer Accuracy – students will see how small errors can drastically affect their results. This is done using an MS Excel[®] spreadsheet to measure how a small angle error in their course will change their final landing point.
- Sextant Solution – students will see how computers can help us work more effectively with complex calculations. Using MS Excel[®], the students will be able to compute detailed corrections for a sextant measurement.

Lesson Opening Topics / Motivation —

How Close is Close Enough

Often, when doing activities, how well we do them is very important. For example, if you are playing baseball and your teammate is not good at throwing you the ball, you might have to run to get the ball. In fact, if you are playing a game, your team might lose a couple of runs. The difference between where your teammate was supposed to throw the ball (to your glove) and where they actually threw the ball (somewhere in left field) is called the *error*. When is error too much? (Possible answer: each situation is different, it depends on the result of the error.) If your

friend throws the ball to you, how much error is okay, and when does the error get large enough to be a problem? (Possible answer: when the throw is off enough that you cannot reach the ball.) We will look at this in Activity 1.

Navigation, Math, and Computers

Even when you think you know where you are going and are not actively using math, you are probably using simple geometric concepts in your head. Mathematical concepts are inherent in navigation, but writing down equations or using a computer to do math is not usually necessary in our everyday lives. In some cases, though, we need to find places that are very far away, we need to find a spot as quickly as possible, or we need to be exact about a location. Doing the math in our heads or on paper can take a long time. Fortunately, computers can help — as you will see in Activity 2.

Major improvements to navigation initially involved creative mathematical thinking and engineering. Before computers existed, engineering was the prime tool to make navigation easier. The *sextant* used angled mirrors, graduated scales, and sturdy materials to make astronomical measurements much more accurate than previous methods. The sextant is a convenient, reliable, and accurate tool still used today. Even the sextant can use the assistance of computers when they are available, as demonstrated in Activity 3.

Things to Remember when Using Computers

Currently, we take computers and calculators for granted. We use them to help us with our homework, play games and even talk to friends. It was only 25 years ago that buying a calculator cost over \$500, and desktop computers, like the ones in your classrooms or homes, did not even exist.

Before that time, the math needed to perform accurate navigation had to be done either by hand or by looking up information in tables. This task can be very tedious and is prone to errors. Today, luckily, we have computers and calculators to help us with math. Computers are really fast at figuring out calculations and repeating tasks, but we still need to understand how the math works so that we can efficiently use the computer. Electrical engineers and computer scientists write the programs that help us in our daily use of computers. For example, an electrical engineer designed the activities you will do today. True, the computer will help you, but you still need know how things work so that you can tell the computer what to do and verify that your results make sense.

Lesson Desired Student Outcomes —

Students will understand that mathematical concepts are required when navigating. Computers are useful for this because they calculate more quickly. Computers allow insight into equations because variables can be repeatedly changed, and the affect on the solution is seen immediately.

Science: Students should be able to:

- Predict (hypothesize). (1)
- Use protractors to measure angles. (1)
- Use meter sticks to measure metric lengths. (1)

- Characterize how a moving object is affected by the moon's gravitational force. (2)
- Describe the advantages as well as disadvantages of using a computer. (5)
- Describe how a computer can be used to help people get to the moon. (5)
- Describe how people used tabulating machines to determine the US census. (5)

Math: Students should be able to:

- Use numbers to count, measure, label, and indicate distances on a map. (1)
- Describe the difference between accuracy and precision. (5)
- Recognize basic trigonometric functions. (4)
- Demonstrate the meaning of square roots. (1)
- Analyze functional relationships. (2)

Colorado State Standards Met

<http://www.mcrel.org/compendium/search.asp>

- Science Standard 1, 2, 5
- Math Standard 1, 2, 4, 5

Lesson Background & Concepts for Teachers —

Accuracy and Precision in Measurement

There are two basic concepts to any collection of measurements, *accuracy* and *precision*. Accuracy is how close the measured data is to the actual value. For example, if you know that you have a mass standard that is 50.00 grams, and you measure it to be 49.98 grams, then your measurement is very accurate. On the other hand, if you measure it to be 43.24 grams, your measurement is not very accurate.

Precision is how close your measurements are relative to each other, not the actual value. For example, let say you measured that same mass standard and came up with the values:

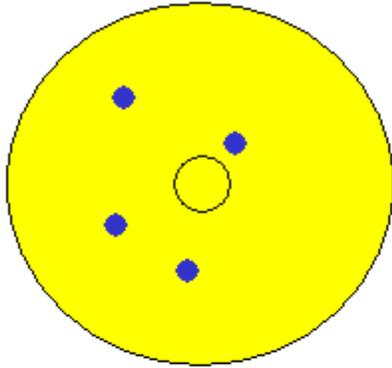
43.24, 43.30, 43.20, 43.25, 43.32

You could say that your measurement was very precise because all the values are very close to each other. On the other hand if your measurements were:

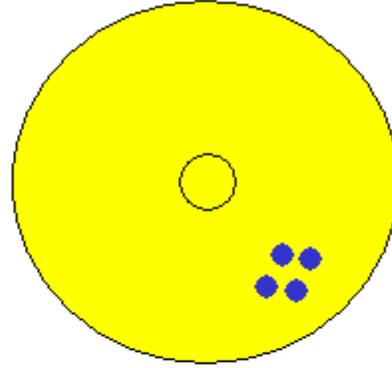
50.03, 43.40, 53.01, 47.54, 30.23

Your measurements would not be very precise because they are all far apart relative to each other.

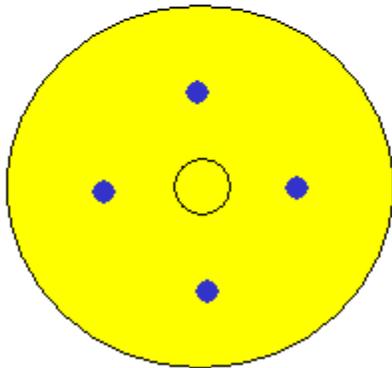
A dartboard is a common example of showing the difference between accuracy and precision (please see dartboard diagrams below). Let's say that someone throws 4 darts at a dartboard. If they are far apart and unevenly spaced, they have no precision *or* accuracy (upper left). The closer the darts are to the center, the better accuracy they have. The closer the grouping of the darts are, the better precision they have (upper right). If the darts are evenly spread around the center, they have low precision but high accuracy because they are not close to each other but all of the darts average to equal the center (lower left). And finally, if you have all your darts closely grouped in the center you have both high accuracy *and* precision (lower right).



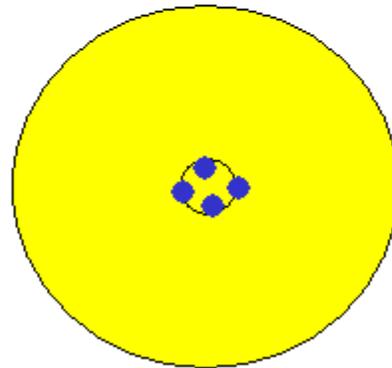
Neither accurate or precise



Precise but not accurate



Accurate but not precise



Accurate and Precise

Diagram created by: Matt Lippis, University of Colorado, Boulder

Accuracy of measurement in navigation depends very much on the situation. If a sailor's target is an island 200 km wide, being off center by 10 or 20 km is no problem. But, if the island were only 1 km wide, it would be missed if off center at all! Many of the measurements made in navigation involve angles, and a small error in the angle can become a large error in position when traveling long distances.

Tabulating Machines

In the 1880s, it was estimated that the next American census in 1890 would need tally the data from more than 62 million Americans. This was not only expensive, but the existing system of tallying marks in small squares on rolls of paper and then adding the marks together by hand was extremely time consuming. Workers realized that continuing to take the census with this method would be impossible and that, somehow, the system would have to be automated. An American inventor, named Herman Hollerith, developed the solution to this problem.

Cards punched by a tram conductor's ticket punch were fed into a mechanism that could read the presence of holes in each card by using spring-mounted nails that passed through the holes to make electrical connections. An electrical connection would move a clock-like counter that accumulated the results!

In addition to solving the census problem, Hollerith's machines were useful for a wide variety of statistical applications. Subsequently, these same techniques were significant in the development of the computer. In February 1924, Hollerith's company changed its name and continued to grow; it is still around today. It is *very* unlikely you have not heard of it: *International Business Machines*. Doesn't ring a bell? Perhaps you know it by its initials: IBM.

Old and New Techniques

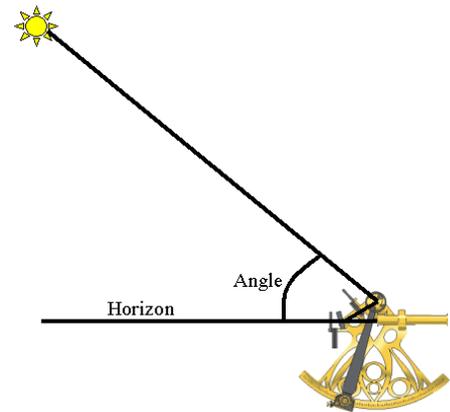
The earliest explorers did not have computers or satellites to help them know their exact location. The most accurate tool developed was the *sextant*, used to determine latitude and longitude. Although sextants are still used today, even they can benefit from computers. The basic use of a sextant is described here, but the concept of its accuracy will be looked at in detail in Activity 2.

How to Use a Sextant

The sextant is a complex device, but luckily it is very easy to use. Basically, a sextant measures the angle between two objects, with at least one being celestial. (Remember, never look directly at the sun! Shields and filters on sextants protect against eye injuries.) The user looks through an eyepiece and, after adjustments, sees both objects superimposed on each other. The measurement of the angle is then taken through the use of two mirrors.

The user looks through the sextant's eyepiece directly at the first object (usually the horizon). There is actually a mirror in the line of sight, but because it is only partially silver, the user can see beyond it (to the horizon). The other mirror is connected to a movable arm and directs light from the second object (usually the sun or moon) and reflects it into the half-silvered mirror. The movable arm is adjusted until you can see both the first object (the horizon) and the second object (the sun or moon). Then you read the measurement of the angle located on the movable arm.

The most common way of using a sextant is to measure the angle between the horizon and the sun. By knowing this angle and your local time you can determine what your latitude is. If you also know the Greenwich Mean Time (GMT), you can also determine your longitude. There are



many things you can determine from a sextant but we will use it to determine your latitude for this exercise. Follow the steps below to use the sextant:

1. Do this exactly at noon, when the sun is at the highest point during the day.
2. Look through the eyepiece.
3. Adjust the angle of the sextant until you see the horizon.
4. Now adjust the movable arm until you see the sun. Because you should never look at the sun directly, the sextant has a number of filters that will block out most of the sun's light, but will let enough through to still see it. Make sure to always use the filters so that you do not hurt your eyes.
5. Adjust the arm until the sun rests just on the horizon.
6. Now measure the altitude of the sun by reading the angle on the movable arm.
7. Write down the exact local time.

Optional for Determining Longitude: Write down the exact GMT (Greenwich Mean Time).

This gives you all the information you need to find your latitude and longitude. Knowing that the measurement was taken when the sun was at its highest point in time, you know that it was noon. This, and the angle of the sun off of the horizon, can tell you your latitude. If you also did Step 7, you could determine your longitude. Knowing the GMT time and the time where you are tells you the difference in time between your location and Greenwich, England (zero degrees longitude). You can relate this time difference to the distance and find your longitude. To actually compute your latitude and longitude, you need to do a number of long calculations — another way that computers can help.

Lesson Vocabulary List —

- **Geometry** – The mathematical relationships of points, lines, angles, surfaces, and solids.
- **Trigonometry** – The mathematical relationships between the sides and the angles of triangles.
- **Sextant** – A navigational instrument containing a graduated 60-degree arc, used for measuring the altitudes of celestial bodies to determine latitude and longitude.
- **Accuracy** – a measure of how close the data comes to the correct answer.
- **Precision** – a measure of how close the data points are relative to each other. In other words, how well grouped your data set is.
- **Computer** – A programmable device that processes information.
- **Equation** – A statement usually written as symbols that are separated into left and right sides and joined by an equal sign.
- **Altitude** – The angle between two points. For example, the angle between the horizon (on the ground) and the sun.

Activity Attachments —

[Activity 1: Close Enough](#) – The concept of accuracy in measurement is introduced.

[Activity 2: Computer Accuracy](#) – Students use an excel spreadsheet to understand how the accuracy of measurements effects navigation.

[Activity 3: Sextant Solutions](#) – Students use an excel spreadsheet to facilitate complicated corrections for a sextant measurement.

Suggestions to Scale Activity for Grades 6 to 8 —

- 6th Grade: Do Activities 1 and 2.
- 7th & 8th Grade: Do Activities 2 and 3.

Lesson Closure and Follow-up —

Computers are taken for granted today. They are smaller and pack more power every year. Understanding how computers fit into the world is important even if you do not want to know how they work. Recognizing parts of navigation that are facilitated by computers is a step in the right direction, and the advantages were seen in these activities. It is also important to recognize the importance of learning and understanding the mathematical concepts that precede the computing. Only when understanding how right triangles can be used, when curved lines can be considered flat, and when to keep a curved line to increase accuracy can you use a computer to its fullest advantage.

Lesson Extension Activities —

- For students with web access, the following PBS site has a great shockwave-based interactive story that lets you try simplified sextant readings and learn about the amazing survival of Shackleton and his crew:
<http://www.pbs.org/wgbh/nova/shackleton/navigate/escape.html>
- Have students research the story of Apollo 13 and perform the calculations — without computers! — that the astronauts had to do in flight to save their lives.
- Using the knowledge from the “Dip of the Sea” portion of Activity 2, have students check the theory by measuring their desk’s latitude as in Lesson 2, Activity 2: North Wall Star. This time they should measure the angle all the way from the floor-wall junction up to the “North Star.” How far off are these new measurements from those taken without the “dip?”

Lesson Assessment and Evaluation —

Pre-Lesson Assessment

- Discussion Question:
 - Computers are everywhere, but do we really need computers? Do you need a computer to tie your shoe? Can a computer help you brush your teeth? Are computers required for navigation? What do you think?

Post-Introduction Assessment

- Voting: Ask a true/false question and have students vote by holding thumbs up for true, and thumbs down for false. Count the number of true and false, and write the number on the board. Give the right answer.
 - True or False: With enough time and supplies, you could navigate anywhere in the world without the help of a computer. (Answer: True, but you would have to have a lot of time.)

Post-Lesson Assessment

- Voting: Ask a true/false question and have students vote by holding thumbs up for true, and thumbs down for false. Count the number of true and false and write the number on the board. Give the right answer.
 - Computers are not absolutely necessary for navigation, but after these activities, who thinks GPS is a good idea?
- Bingo: Provide each student with a sheet of paper containing a list of the lesson vocabulary terms. Have each student walk around the room and find a student who can define one vocabulary term. Students must find a different student for each term. When a student has all terms completed s/he shouts “Bingo!” Continue until all students bingo.

Homework

Internet Search: Have students research a few of the concepts from the lesson on the Internet and lead a brief discussion of the findings during the next class period.

Lesson References —

Other interesting information you might like to look at:

<http://www.nps.gov/jeff/LewisClark2/Education/ForTeachers/PatrickGassJourney/GassLtrPg21.htm>

Sextant Technique

<http://www.sailnet.com/sailing/97/tecjan97.htm>

<http://www.dade.k12.fl.us/lifeskills/crew/sextants.html>

<http://www.techdirections.com/TEP2pro15.pdf>

http://www.scsc.k12.ar.us/1999outwest/members/MockK/lesson_plan.htm

Treasure Hunt: (navigation graphics)

<http://academic.brooklyn.cuny.edu/geology/leveson/core/linksa/treasurehunt.html>

History of the Sextant

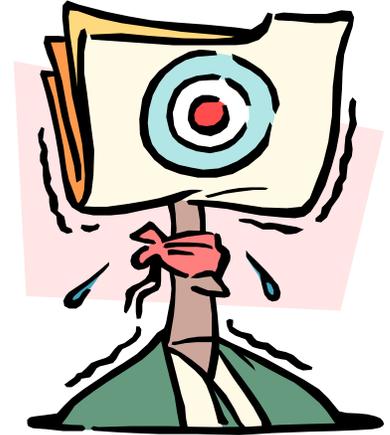
<http://home.earthlink.net/~nbrass1/cardart.htm>

Activity: Close Enough?

This activity is planned for 28 students working both individually and in groups of two.

Activity Materials List —

- 28 [Accuracy By Hand Worksheets](#)
- 1 Protractor (or print out [paper half-protractors](#))
- 1 piece of string or rope (exactly 10 meters long – use one color to mark the zero point and other colors to mark the 6 and 8 meter points as well)
- Small Dixie cups for markers (if available or you can use masking tape)
- Masking tape



Activity Equipment and Tools List —

- Calculators (not absolutely necessary but helpful)
- [Trig Tables](#) (if calculators are not available)
- Pencils
- Tape measure or meter stick.

Activity Cost Estimate —

\$0

Activity Time Estimate —

40-50 min.

Activity Procedure —

A. Background:

Accuracy and Precision in Measurement

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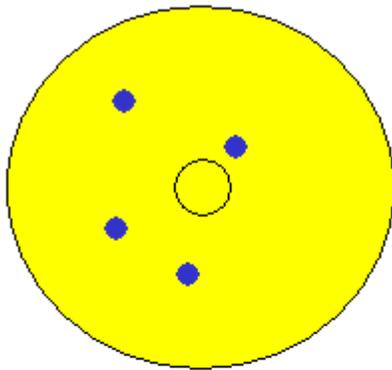
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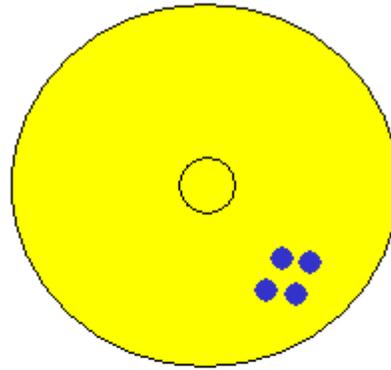
50.03, 43.40, 53.01, 47.54, 30.23

your measurements wouldn't be very precise, because they are all far apart relative to each other.

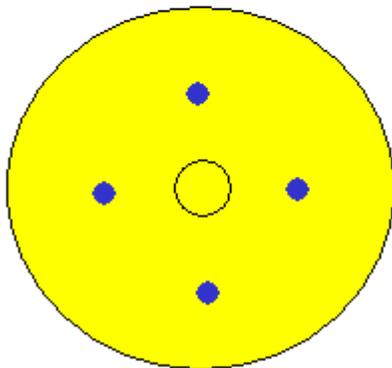
A common example of the difference between accuracy and precision is looking at a dartboard. Let's say that someone throws 4 darts at a dartboard. The closer the darts are to the center, the better accuracy they have. The closer the grouping of the darts are, the better precision they have (upper right). If your darts are evenly spread around to the center, they have low precision but high accuracy because they are not close to each other but all of the darts average to equal the center (lower left). And finally, if you have all your darts closely grouped in the center you have both high accuracy and precision (lower right).



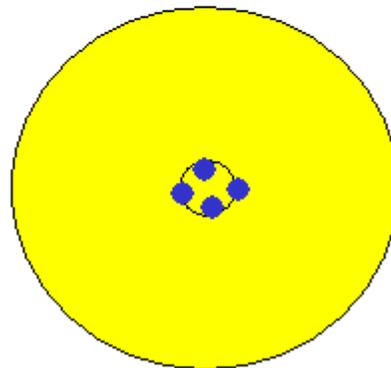
Neither accurate or precise



Precise but not accurate



Accurate but not precise

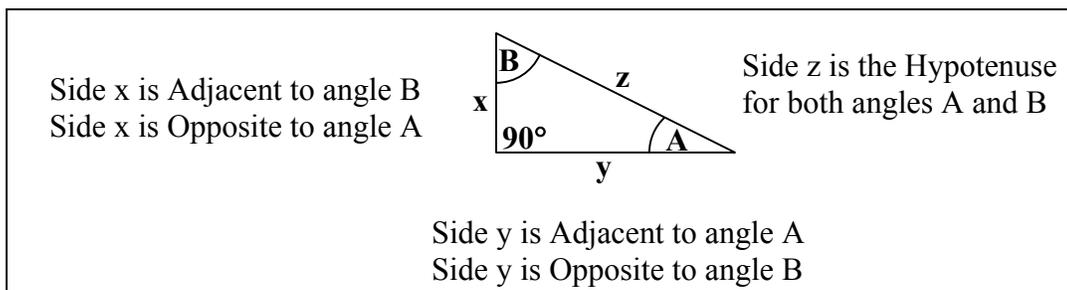


Accurate and Precise

Accuracy of measurement in navigation depends a lot on the situation. If a sailor's target is an island 200 km wide, being off of the center by 10 or 20 km is no problem. But if the island is only 1 km wide, the sailor would miss the island all together. Many of the measurements made in navigation involve angles, and a small error in the angle can become a large error in position when traveling long distances.

Trigonometry Review

Trigonometry is a branch of mathematics dealing with relationships of the angles and sides of triangles. The three basic trigonometric relations that we are concerned with here are: sine, cosine, and tangent (abbreviated as sin, cos, and tan). They are ratios of the lengths of two sides of a particular triangle. The particular type of triangle that is most useful in mathematics is the “right” triangle, which has one angle equal to 90 degrees. By definition, the 90-degree angle is made by two lines that are perpendicular to each other (like the corner of a square) and third side of the triangle is made by a sloping line connecting the two perpendicular lines. This sloping line is called the hypotenuse and the name comes from the Greek *hypo* (meaning under) and *teinein* (meaning to stretch). It is easiest to show this visually:



<p><u>For Angle “A”:</u></p> $\sin(A) = \frac{\text{Opposite}}{\text{Hypotenuse}} = \frac{x}{z}$ $\cos(A) = \frac{\text{Adjacent}}{\text{Hypotenuse}} = \frac{y}{z}$ $\tan(A) = \frac{\text{Opposite}}{\text{Adjacent}} = \frac{x}{y}$	<p><u>For Angle “B”:</u></p> $\sin(B) = \frac{\text{Opposite}}{\text{Hypotenuse}} = \frac{y}{z}$ $\cos(B) = \frac{\text{Adjacent}}{\text{Hypotenuse}} = \frac{x}{z}$ $\tan(B) = \frac{\text{Opposite}}{\text{Adjacent}} = \frac{y}{x}$
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SOH CAH TOA. These letters can help students remember which sides go with which functions (Sine = Opposite / Hypotenuse, etc.). Mnemonics may help 6-8th graders to memorize the relations: “Some Old Hag Caught A Hippie Tripping On Art”; or “Some Oaf Happily Cut A Hole Through Our Apartment”. SOH CAH TOA.

A navigation example using trigonometry is shown here:

<http://www.staff.vu.edu.au/mcaonline/units/trig/ratios.html>

B. Before the Activity

1. Print out the worksheets, trig tables, and paper protractors if needed.
2. Decide whether activity will be performed indoors or outside. You need a space that is at least a 10 meter square of floor/ground space.
3. Measure and mark string or rope (see materials list).

C. With the Students:

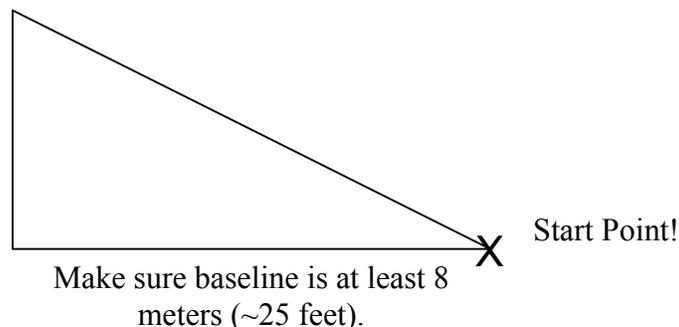
In the classroom:

Ask the students how accurately things have to be measured. Should everything be measured down to the millimeter? Can we round off all measurements to the nearest foot? Explain that the needed accuracy of a measurement depends on what is being measured. When navigating, how accurately do we need to measure an angle? Let's investigate:

1. Give each student an [Accuracy By Hand Worksheet](#).
2. Have students use calculators or Trig Table sheets to do the Sailing to an Island problem.
3. Have students use calculators or Trig Table sheets to do the Rocket to the Moon problem.
4. Help students answer the question on the bottom of the worksheet: Why must the rocket be more accurate? A: Because the target is smaller relative to the distance traveled than the island is. This can be seen by looking at the ratio of the size of the target and the distance to it: Island = $50/1000 = .05$ and Moon = $1738/382700 = .0045$.

Now the students will try this in real life. Explain that you have a navigation mission for them that requires accuracy of measurement. They are going to try and locate an Invisible Island!

5. Divide class into teams of two and hand out markers (small cups with tape on them so students can mark their names or just pieces of tape will work as well. If outside, some sort of weight should be put in cup – gram weights, rocks, etc.)
6. Take students to the test area (field outside, gym, or large enough classroom).
7. Choose a natural straight line (edge of field, sidewalk, line in gym, or even a wall will work) that is at least 8 meters long (25 feet) and mark the start point with an "X" of tape or object that can be stuck into the ground and place the protractor on the start point. (See below)



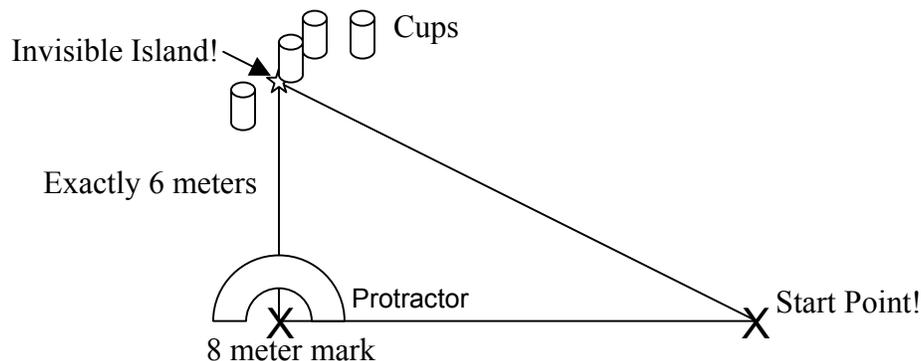
Tell students that the Invisible Island is located exactly 10 meters away and 37° off of the baseline from the start point.

Note: Students must stay in line to have their turn – those getting out of line should be sent to the back.

8. One partner will line up the protractor and hold the zero end of the string at its center point. The other partner, carrying the team cup, will take the end of the string and walk out at a 37° angle until the rope is tight. When both partners think they are close to having the final position at a 37° angle from the baseline, the cup is set down.
9. Have each group place their cup where they think the island is, but caution each group not to just put their cup by another groups, they may be wrong!

Once all groups have placed their cups it's time to reveal the location of the Invisible Island:

10. Use the marked string to measure exactly 8 meters along the baseline from the start point and mark this spot.
11. Use the protractor to measure a 90° angle at this spot.
12. Use the string to measure exactly 6 meters along this line (see below: perpendicular to the baseline toward all the cups) and mark this spot as the island.



13. Use a meter stick or tape measure to measure the distance of each cup from the actual island location and write the distance on the tape on the cup.
14. Have the teams return to the classroom with their cups.
15. Have the students look at the [Accuracy By Hand Worksheet](#) again and notice that the numbers for the “Sailing to an Island” problem are similar to the ones just used: Instead of the island being 1000 km away, the invisible island was 1000 cm

away (there are 100 cm in a meter so $10 \times 100 = 1000\text{cm}$). From this information, have them estimate how far off their angle measurement was.

(For ambitious students: work the equation backward to find how far off the angle they were. This will require using the ‘arcsin’ or ‘inverse sin’ function on a calculator.)

Math Skills Reinforced —

6th, 7th, 8th: Students use right triangle trigonometry and angle measurements to calculate distances. Students will also explore accuracy and precision

Activity Troubleshooting Tips —

This activity requires a large open space to complete. You will need at least a 10m square of ground or floor to work in. If not, the students may run into a wall or desk and not be able to find the “mystery island.”

Activity Desired Student Outcomes —

After this activity, students should understand a mistake in measuring an angle could make a huge difference if the target is far away. The needed accuracy of a measurement depends on what is being measured.

Activity Assessment & Evaluation —

Pre-Activity Assessment

- Discussion Question: Solicit, integrate, and summarize student answers.
 - Ask the students how accurately things have to be measured. Should everything be measured down to the millimeter? Can we round off all measurements to the nearest foot? (Answer: Explain that the needed accuracy of a measurement depends on what is being measured. See Part C, *With the Students*.)

Activity Embedded Assessment

- Worksheet: Accuracy calculations: Students follow and complete the worksheet.

Post-Activity Assessment

- Questions/Answers:
 - Why was it hard to get the angle right to find the Invisible Island? (Answer: Many possible reasons: the string or rope was much thicker than the divisions on the protractor, the protractor only had divisions of 5 degrees, it was hard to get the string tight, it was hard to keep the protractor lined up, etc.)
 - What would need to be done to get close to a spot a kilometer away? (Answer: A more accurate angle measurement would need to be made, the easiest would be to use something like a 5-foot long protractor that showed divisions down to the 1/100 of a degree. Finding a rope this long would be a problem though.)

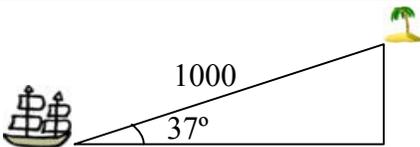
Suggestions to Scale Activity for Grades 6 to 8 —

6th Grade: Use or make a fairly large protractor (photocopier enlargement is an option).

7th Grade: Do as is.

8th Grade: Have students use a compass to stay on the correct angle until the string is tight.

The Importance of Accurate Measurements

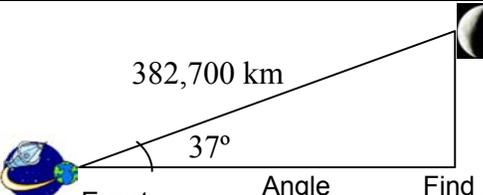


Sailing to an Island

Name: _____

Date: _____

Angle off by: (degrees)	Exact Angle	Angle mistakenly used	Find sine of Angle (use trig table or a calculator)	Mult by 1000km	Length of Side Opposite Angle	Subtract Exact Value	Final distance off by: (km)	Within 50km of the island center?			
5.0	+	37.0	=	>	*	1000.0	=	-	600.0		
2.0	+	37.0	=	>	*	1000.0	=	-	600.0		
0.0	+	37.0	=	>	*	1000.0	=	-	600.00	0.0	Yes
-2.0	+	37.0	=	>	*	1000.0	=	-	600.0		
-5.0	+	37.0	=	>	*	1000.0	=	-	600.0		

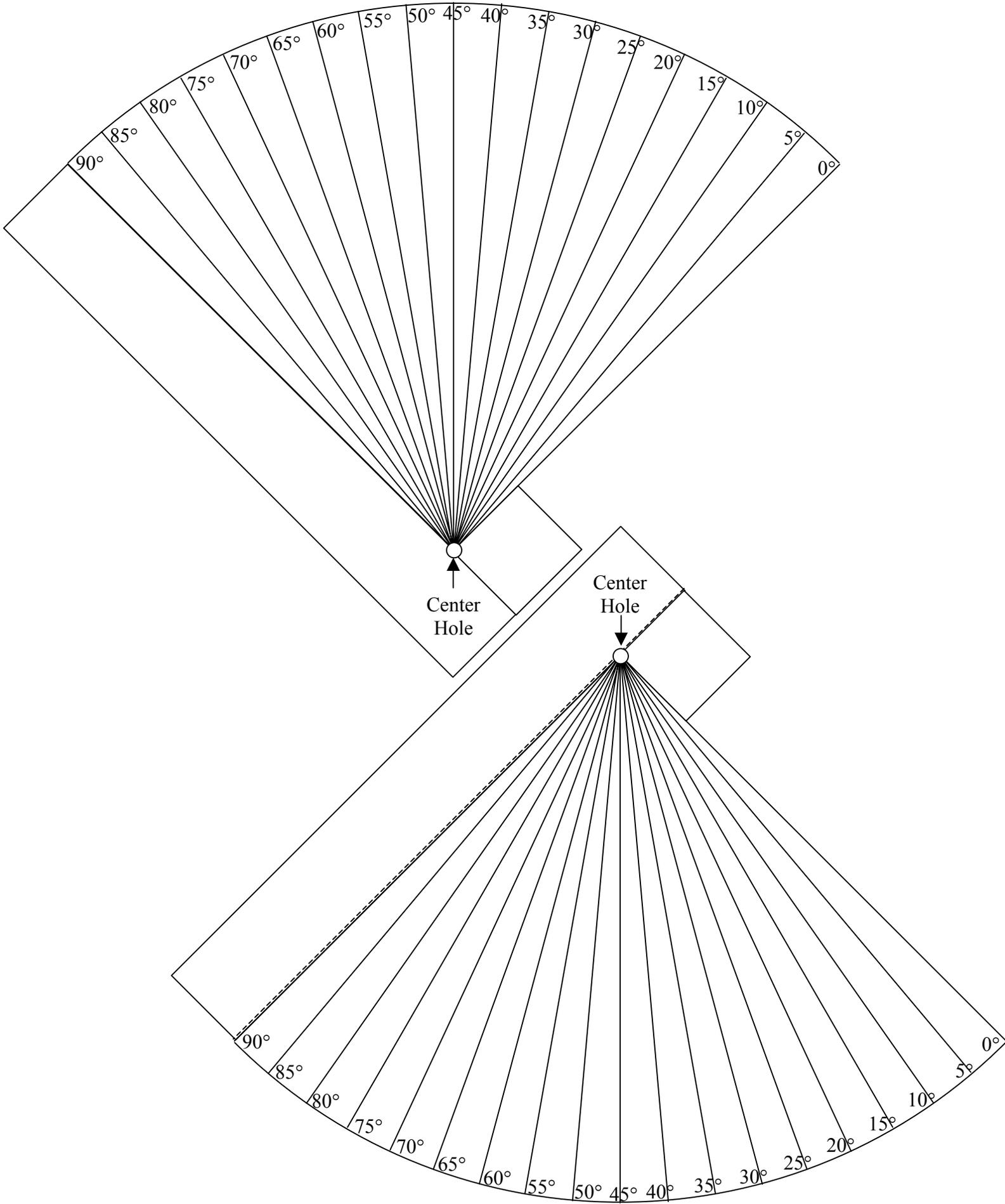


Rocket Ship to the Moon

Angle off by: (degrees)	Exact Angle	Angle mistakenly used	Find sine of Angle (use trig table or a calculator)	Mult by 382,700 km	Length of Side Opposite Angle	Subtract Exact Value	Final distance off by: (km)	Within 1738 km of the Moon center?			
1.0	+	37.0	=	>	*	382700.0	=	-	230314.6		
0.2	+	37.0	=	>	*	382700.0	=	-	230314.6		
0.0	+	37.0	=	>	*	382700.0	=	-	230314.6	0.0	Yes
-0.2	+	37.0	=	>	*	382700.0	=	-	230314.6		
-1.0	+	37.0	=	>	*	382700.0	=	-	230314.6		

Why must the rocket be more accurate? _____

Half Protractors:



Navigation Trigonometry Tables

Angle (deg)	Sine	Cosine	Tangent		Angle (deg)	Sine	Cosine	Tangent
0	0.0000	1.0000	0.0000		Continued			
1	0.0175	0.9998	0.0175		46	0.7193	0.6947	1.0355
2	0.0349	0.9994	0.0349		47	0.7314	0.6820	1.0724
3	0.0523	0.9986	0.0524		48	0.7431	0.6691	1.1106
4	0.0698	0.9976	0.0699		49	0.7547	0.6561	1.1504
5	0.0872	0.9962	0.0875		50	0.7660	0.6428	1.1918
6	0.1045	0.9945	0.1051		51	0.7771	0.6293	1.2349
7	0.1219	0.9925	0.1228		52	0.7880	0.6157	1.2799
8	0.1392	0.9903	0.1405		53	0.7986	0.6018	1.3270
9	0.1564	0.9877	0.1584		54	0.8090	0.5878	1.3764
10	0.1736	0.9848	0.1763		55	0.8192	0.5736	1.4281
11	0.1908	0.9816	0.1944		56	0.8290	0.5592	1.4826
12	0.2079	0.9781	0.2126		57	0.8387	0.5446	1.5399
13	0.2250	0.9744	0.2309		58	0.8480	0.5299	1.6003
14	0.2419	0.9703	0.2493		59	0.8572	0.5150	1.6643
15	0.2588	0.9659	0.2679		60	0.8660	0.5000	1.7321
16	0.2756	0.9613	0.2867		61	0.8746	0.4848	1.8040
17	0.2924	0.9563	0.3057		62	0.8829	0.4695	1.8807
18	0.3090	0.9511	0.3249		63	0.8910	0.4540	1.9626
19	0.3256	0.9455	0.3443		64	0.8988	0.4384	2.0503
20	0.3420	0.9397	0.3640		65	0.9063	0.4226	2.1445
21	0.3584	0.9336	0.3839		66	0.9135	0.4067	2.2460
22	0.3746	0.9272	0.4040		67	0.9205	0.3907	2.3559
23	0.3907	0.9205	0.4245		68	0.9272	0.3746	2.4751
24	0.4067	0.9135	0.4452		69	0.9336	0.3584	2.6051
25	0.4226	0.9063	0.4663		70	0.9397	0.3420	2.7475
26	0.4384	0.8988	0.4877		71	0.9455	0.3256	2.9042
27	0.4540	0.8910	0.5095		72	0.9511	0.3090	3.0777
28	0.4695	0.8829	0.5317		73	0.9563	0.2924	3.2709
29	0.4848	0.8746	0.5543		74	0.9613	0.2756	3.4874
30	0.5000	0.8660	0.5774		75	0.9659	0.2588	3.7321
31	0.5150	0.8572	0.6009		76	0.9703	0.2419	4.0108
32	0.5299	0.8480	0.6249		77	0.9744	0.2250	4.3315
33	0.5446	0.8387	0.6494		78	0.9781	0.2079	4.7046
34	0.5592	0.8290	0.6745		79	0.9816	0.1908	5.1446
35	0.5736	0.8192	0.7002		80	0.9848	0.1736	5.6713
36	0.5878	0.8090	0.7265		81	0.9877	0.1564	6.3138
37	0.6018	0.7986	0.7536		82	0.9903	0.1392	7.1154
38	0.6157	0.7880	0.7813		83	0.9925	0.1219	8.1443
39	0.6293	0.7771	0.8098		84	0.9945	0.1045	9.5144
40	0.6428	0.7660	0.8391		85	0.9962	0.0872	11.4301
41	0.6561	0.7547	0.8693		86	0.9976	0.0698	14.3007
42	0.6691	0.7431	0.9004		87	0.9986	0.0523	19.0811
43	0.6820	0.7314	0.9325		88	0.9994	0.0349	28.6363
44	0.6947	0.7193	0.9657		89	0.9998	0.0175	57.2900
45	0.7071	0.7071	1.0000		90	1.0000	0.0000	Infinity
Angle (deg)	Sine	Cosine	Tangent		Angle (deg)	Sine	Cosine	Tangent

Activity: Computer Accuracy

This activity is planned for 28 students working in groups equal to the number of computers available.

Activity Materials List —

- Computer Accuracy Excel File
- [Computer Accuracy Worksheet](#)

Activity Equipment and Tools List —

- Computers with Microsoft Office Excel
- Pencils



Activity Cost Estimate —

\$0 (assuming computers are available)

Activity Attachments —

- Computer Accuracy Excel File
- [Computer Accuracy Worksheet](#)

Activity Time Estimate —

40-50 min.

Activity Procedure —

A. Background

Accuracy and Precision in Measurement

There are two basic concepts to any collection of measurements, accuracy and precision. *Accuracy* is how close the measured data is to the actual value. For example, if you know that you have a mass standard that is 50.00 grams and you measure it to be 49.98 grams, your measurement is very accurate. On the other hand, if you measure that mass to be 43.24 grams, your measurement is not very accurate.

Precision is how close your measurements are relative to each other, not the actual value. For example, let say you measured that same mass standard and came up with the values:

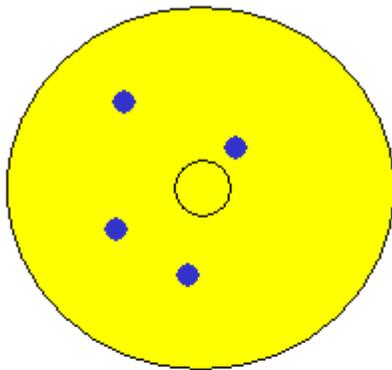
43.24, 43.30, 43.20, 43.25, 43.32

you could say that your measurement was very precise. This is because all the values are very close to each other. On the other hand if your measurements were:

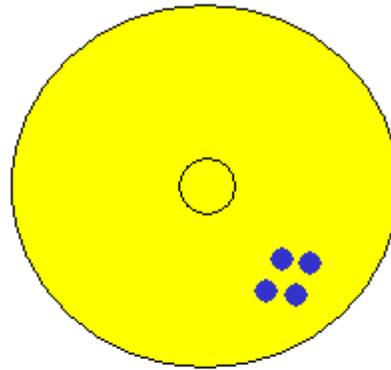
50.03, 43.40, 53.01, 47.54, 30.23

your measurements would not be very precise, because they are all far apart relative to each other.

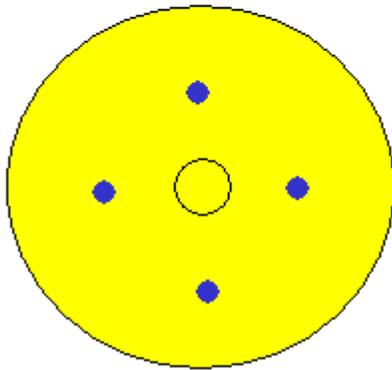
A common example of the difference between accuracy and precision is looking at a dartboard. Let's say that someone throws 4 darts at a dartboard — The closer the darts are to the center, the better accuracy they have. The closer the grouping of the darts are, the better precision they have (upper right). If your darts are evenly spread around the center, they have low precision but high accuracy because they aren't close to each other but all of the darts average to equal the center (lower left). And finally, if you have all your darts closely grouped in the center you have both high accuracy and precision (lower right).



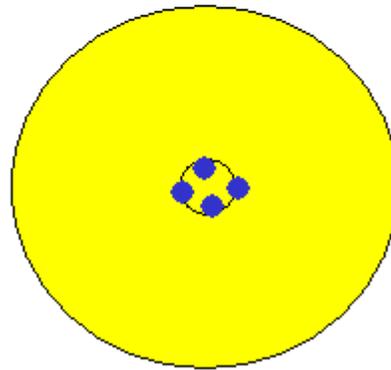
Neither accurate or precise



Precise but not accurate



Accurate but not precise

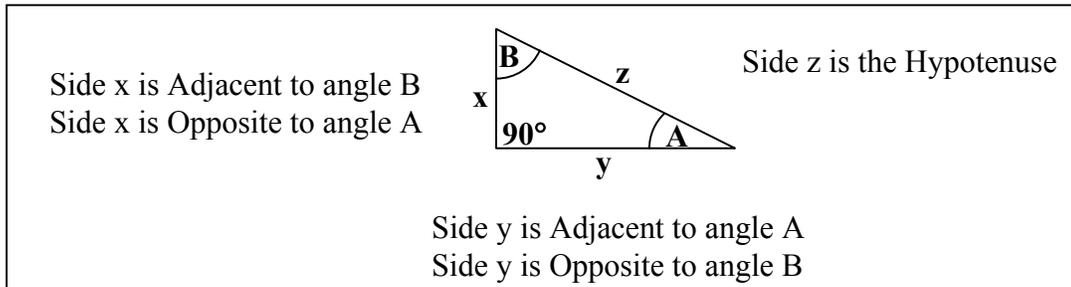


Accurate and Precise

Accuracy of measurement in navigation depends a lot on the situation. If a sailor's target is an island 200 km wide, being off of the center by 10 or 20 km is no problem. But if the island is only 1 km wide, the sailor would miss the island all together. Many of the measurements made in navigation involve angles – and a small error in the angle can become a large error in position when traveling long distances.

Trigonometry Review

Trigonometry is a branch of mathematics dealing with relationships of the angles and sides of triangles. The three basic trigonometric relations that we are concerned with — sine, cosine and tangent — are ratios of the lengths of two sides of a particular triangle. A very useful type of triangle is a *right* triangle, which has one angle equal to 90 degrees. By definition, the 90° angle is made by two lines that are perpendicular to each other (like the corner of a square), and the third side of the triangle is made by a sloping line connecting the two perpendiculars. This sloping line is called the *hypotenuse*, and the name comes from the Greek words *hypo* (meaning *under*) and *teinein* (meaning *to stretch*). Essentially, hypotenuse means *to stretch under the 90° angle*. It is easiest to show this visually.



<u>For Angle "A":</u>	<u>For Angle "B":</u>
$\sin(A) = \frac{\text{Opposite}}{\text{Hypotenuse}} = \frac{x}{z}$	$\sin(B) = \frac{\text{Opposite}}{\text{Hypotenuse}} = \frac{y}{z}$
$\cos(A) = \frac{\text{Adjacent}}{\text{Hypotenuse}} = \frac{y}{z}$	$\cos(B) = \frac{\text{Adjacent}}{\text{Hypotenuse}} = \frac{x}{z}$
$\tan(A) = \frac{\text{Opposite}}{\text{Adjacent}} = \frac{x}{y}$	$\tan(B) = \frac{\text{Opposite}}{\text{Adjacent}} = \frac{y}{x}$

SOH CAH TOA. These letters can help students remember which sides go with which functions (Sine = Opposite / Hypotenuse, etc.). Mnemonics may help 6-8th graders to memorize the relations: "Some Old Hag Caught A Hippie Tripping On Art"; or "Some Oaf Happily Cut A Hole Through Our Apartment". SOH CAH TOA.

To see a handy navigation example using trigonometry, check out this website:
<http://www.staff.vu.edu.au/mcaonline/units/trig/ratios.html>

B. Before the Activity

1. It is suggested that a trial-run of the activity be performed beforehand.
2. Print out [Computer Accuracy Worksheets](#) for each group or individual (may be printed double sided).
3. The excel files are protected from changes (with the exception of the data entry boxes) but the protection can be removed. If students are fairly computer savvy, a password may be added to guard the protection. If you are going to do this – do it and save *before* loading onto other computers!
 - a. On the **Tools** menu, point to **Protection**, and then click **Unprotect Sheet**.
 - b. On the **Tools** menu, point to **Protection**, and then click **Protect Sheet**.
 - c. When prompted, leave all boxes checked and enter the protection password for the worksheet. Passwords are case sensitive. To unprotect the sheet again, you must type the password exactly as it was created, including uppercase and lowercase letters.
4. Load the Computer Accuracy Excel File onto all computers and put it in an easy to access place (Desktop for instance) or better yet – have it opened up when students arrive.
5. MAKE SURE to start the students on the excel file with answers that are incorrect! If necessary, input “10” into all of the yellow boxes before the students arrive.

C. With the Students

Before students go to the computers:

1. Divide the class into groups (depending on the number of computers available) and give each student or group the [Computer Accuracy Worksheets](#).
2. Discuss the concepts of the “Sail to an Island Harbor” problem in terms of the increased accuracy needed compared to just finding the island. Remind the students that they know the exact measurements from their ship to the center of the island. However, they don’t want to reach the center of the island, they want to find the harbor! **Students will need to adjust the angle so that they are 1 km north of the island.** This will allow the ship to get close enough to the harbor to see it with the naked eye if they are slightly off. What happens if it is foggy outside? **The ship now needs to be within 100m (0.1 km) of the island to be able to see it.** How accurate does the new angle need to be?
3. Discuss the concepts of the “Rocket to a Moon Orbit” – this problem shows how the error in the navigation measurement of the initial angle can greatly affect the final destination. Remind students that they will run this experiment twice. Once with an accuracy error of $\pm 0.1^\circ$, and again with an accuracy error of $\pm 0.01^\circ$. (A new angle of around 37.38° will work well. Students will not be able to launch successfully with a error of $\pm 0.1^\circ$ because all three outcomes are possible with the given error).

At the Computers:

4. Have students do the “Sail to an Island Harbor” problem and come up with creative things to search for on the mystery island..
5. Have students do the “Rocket to a Moon Orbit” problem and answer the questions at the bottom of the Worksheet.
6. Each group or individual should print out the Excel file with the correct angles (assuming printers are available – otherwise students could write their numbers on the worksheet).

(For ambitious students: work the equations backward to find the needed angles without all the guess work – this will require using the ‘arcsin’ or ‘inverse sin’ function on a calculator. Is it faster to solve the equations backward or guess on the computer?)

Math Skills Reinforced —

6th, 7th, 8th: Students will understand the difference between accuracy, precision, and error. They will also recognize basic trigonometric functions and analyze functional relationships.

Activity Troubleshooting Tips —

These worksheets could be confusing if not explained thoroughly from the start. Remind students that in the case of both the ship and the rocket, there are known distances and angles and they have to make corrections to them to reach a target (harbor or orbit).

Activity Desired Student Outcomes —

After this activity, students should understand that Math is fundamental to navigation. Computers are much faster at doing math calculations than humans. When higher accuracy is needed, calculations become more difficult so using computers is even more practical.

Activity Assessment & Evaluation —

Pre-Activity Assessment

- Discussion:
 - (Hold up protractor if you have one) When we sent astronauts to the moon, do you think it mattered if they were off by 1 degree? How about if they were off by 1/10 of a degree – would they still make it? Should we worry if they are off by 1/1000 of a degree? (Answer: Depends on the mission requirements , but have students keep these thoughts in mind during the activity.)

Activity Embedded Assessment

- Worksheet/Computer calculations
 - Students follow and complete the excel file and worksheet.

Post-Activity Assessment

- Questions/Answers for Moon problem:
 - Why can't you get three "yes" boxes when the accuracy is equal to 0.1? A: Because the difference of the final distances calculated by the angle you guessed and its possible error angle is larger than the window of safe arrival.
 - For the example of the moon problem, can the students better answer the pre-activity questions? A: For this case, 1/100 of a degree mattered and 1/1000 did not.

Suggestions to Scale Activity for Grades 6 to 8 —

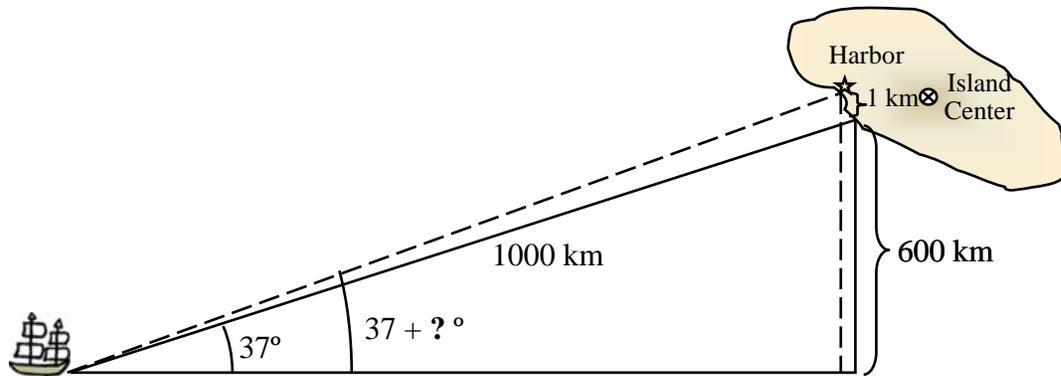
6th Grade: Do as is.

7th Grade: Have the students find the largest accuracy error that still allows a safe launch (i.e. the rocket arrives at exactly 2000 km and has +/- error equal to 99 km).

8th Grade: Have the students do the “Sail to an Island Harbor” by hand first (give them a first guess of 37.1° and tell them they should try to get an opposite side length within 0.1 km of 602.8) and then check their answer with the excel file. Hopefully they will only have to do the calculations three times: 37.1 – see it needs to be smaller. 37.05 – see it needs to be bigger. 37.075 – bingo!

Increasing the Accuracy of Measurements

Sailing to an Island Harbor!



Name: _____

Date: _____

Angle Adjustment (in degrees)	Known Angle	Corrected Angle	Find sine of Angle (use trig table or a calculator)	Mult by 1000km	Length of Side Opposite Angle	Subtract Known Value	Final distance away from Island Center: (km)	Within 0.1 km from the harbor?
?	+ 37.0	= ?	> ?	x 1000.0	= ?	- ?	= 1.0	Yes
0.0	+ 37.0	= 37.0	> 0.60	x 1000.0	= 600.00	- 600.0	= 0.0	No

To do this problem by hand would mean guessing and doing all the calculations multiple times until the proper distance is found. While the calculation is not that difficult, doing it over and over is time consuming. Instead, a computer can be used to do all the calculating (once the equations have been programmed in).

AT THE COMPUTER:

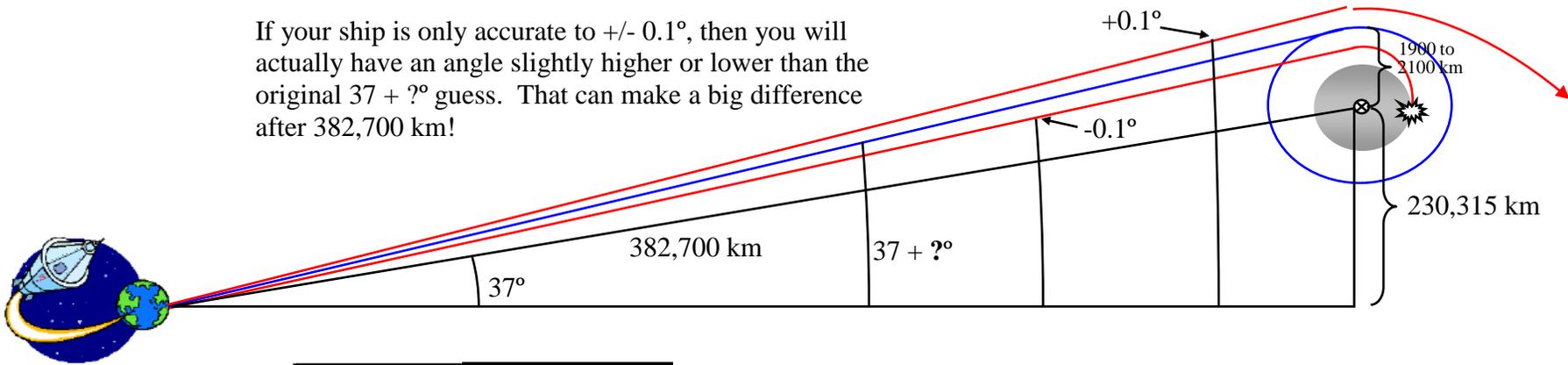
To solve the problem, you simply supply the guesses in the highlighted box → press ENTER → get the calculated result immediately (computers do math much faster than humans). As the captain of the ship, you know that you are exactly 1000 km from the center of the island and at an angle of 37°. You also know that the harbor is 1km **north** of the center. Therefore you need to adjust the 37° angle so that your ship arrives within 1km of the center of the island, **and 0.1km of the harbor!**

Have each person in your group write down something they would search for on this mystery island!:

Increasing the Accuracy of Measurements

Rocket Ship to a Moon Orbit!

If your ship is only accurate to $\pm 0.1^\circ$, then you will actually have an angle slightly higher or lower than the original $37 + ?^\circ$ guess. That can make a big difference after 382,700 km!



Accuracy Error =

Plus and Minus Errors (degrees)	Known Angle	Angle used	Find sine of Angle (use trig table or a calculator)	Mult by 382,700 km	Length of Side Opposite Angle	Subtract Exact Value	Final distance off by: (km)	Roughly 1900 to 2100 km from the Moon center?
<input style="border: 1px solid black; padding: 2px 10px;" type="text" value="0.1"/>	+	37.1	= 37.1 + ?	>	<input style="border: 1px solid black; padding: 2px 10px;" type="text" value="?"/>	x 382700.0 = <input style="border: 1px solid black; padding: 2px 10px;" type="text" value="?"/>	- 230,314.6 km = <input style="border: 1px solid black; padding: 2px 10px;" type="text" value="?"/>	<input style="border: 1px solid black; padding: 2px 10px;" type="text" value=""/>
<input style="border: 1px solid black; padding: 2px 10px;" type="text" value="?"/>	+	37.0	= 37 + ?	>	<input style="border: 1px solid black; padding: 2px 10px;" type="text" value="?"/>	x 382700.0 = <input style="border: 1px solid black; padding: 2px 10px;" type="text" value="?"/>	- 230,314.6 km = 2000.0	Yes
<input style="border: 1px solid black; padding: 2px 10px;" type="text" value="-0.1"/>	+	36.9	= 37.1 - ?	>	<input style="border: 1px solid black; padding: 2px 10px;" type="text" value="?"/>	x 382700.0 = <input style="border: 1px solid black; padding: 2px 10px;" type="text" value="?"/>	- 230,314.6 km = <input style="border: 1px solid black; padding: 2px 10px;" type="text" value="?"/>	<input style="border: 1px solid black; padding: 2px 10px;" type="text" value=""/>

AT THE COMPUTER:

First, your engineers know the distance and the angle from the Earth to the center of the Moon very accurately. You are trying to get your rocket into orbit around the Moon, so you need to adjust your 37° angle. However, you cannot be entirely sure of your new measurements. That means your measurement will be within $\pm 0.1^\circ$. **First:** Enter this number in the highlighted outlined Accuracy Error box. Will this $\pm 0.1^\circ$ make a difference? Try to find the needed adjustment to the 37° angle that puts your rocket in orbit.

Is it possible to launch safely if you can only aim your rocket to within $\pm 0.1^\circ$? _____

What is your best angle? _____

Second: Upgrade your navigation system so it is accurate within $\pm 0.01^\circ$ (enter this number in the Accuracy Error box).

Can you get both the plus and minus errors within the safe zone? _____

Is it possible to launch safely now? _____ What is your best angle? _____

Activity: Sextant Solutions

This activity is planned for 28 students working individually or in groups according to the number of computers available.

Activity Materials List —

- Sextant Corrections Excel File
- [Sextant Corrections Worksheets](#) (The teacher can decide whether each student will be given a worksheet to complete or students will complete one worksheet per group)

Activity Equipment and Tools List —

- Computers with Microsoft Office - Excel
- Pencils
- Calculators (if available)

Activity Cost Estimate —

\$0 (assuming computers are available.)

Activity Attachments —

Sextant Corrections Excel File
[Sextant Corrections Worksheet](#)

Activity Time Estimate —

40-50 min.

Activity Procedure —

A. Background:

The earliest explorers did not have computers or satellites to help them know their exact location. The most accurate tool developed was the sextant to determine latitude and longitude.

Continuing with the concept of trying to measure small angles accurately, introduce the sextant and discuss it with the class. Instead of trying to line up a separate horizon line and sun line, the sextant allowed the two images to be moved together — greatly increasing the accuracy of the angle measured.

A sextant's accuracy is expressed in "seconds of arc." A degree is divided into 60 minutes (noted as 60') and a minute is divided into 60 seconds (noted as 60"). A good thing to remember is that each minute of angular measurement represents a distance of one nautical mile. A sextant scale can generally read out to one-fifth or one-tenth of a minute — that seems pretty accurate! But that reading is not the final accuracy, as there are several corrections that must be made to that angle. At this point the navigator must do



what amounts to a full page of corrections and calculations using astronomical tables and charts. The accuracy of the correction values affects the final result and the calculations offer ample opportunity for human error.

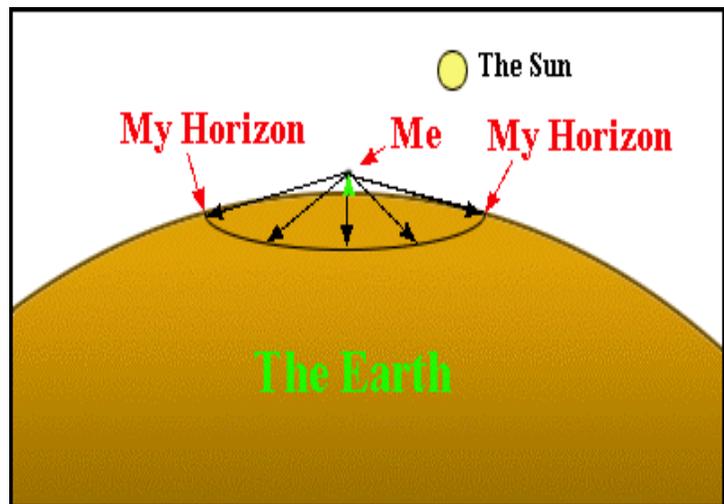
Depending on the accuracy of the corrections, the final angular measurement could easily be off by several minutes or more, so most navigators (assuming they were skilled and had good weather) could expect at best an accuracy of within a few miles.

Today refined manufacturing techniques and robust materials make sextants last longer but cannot improve the accuracy. The accuracy is limited by the fuzzy edges of celestial objects. Skill in use and better understanding of the math and geometry involved can improve accuracy slightly compared to the past. But there is still the large chance of error in the pages of calculations needed! This is where a modern advantage finally comes in — the computer.

When used properly by an experienced navigator and under ideal weather conditions, a well-made sextant can measure an angle with precision to the nearest ten seconds of arc (10 seconds of a degree is about 0.003 degrees of a 360-degree circle!). A computer can do the corrections and calculations quickly and an accuracy of 0.2 miles in final position is possible. More likely it will be about twice that under normal weather conditions (0.4 miles) and in poor conditions it may still be 1-2 miles off. No better than in good weather conditions hundreds of years ago, but thanks to the computer you did not have to do all that math! This activity will show how computers can be used to understand equations even when people don't know how to do the math.

Sextant Use and Error

The *sextant* is a high precision instrument. Be very careful when handling one. Even shaking it roughly might cause damage. The movable arm has an arc range of 60 degrees, and this is why it is called a sextant. You double this measurement to 120 degrees to find your altitude angle. Every sextant has an inherent error, which is called its *offset*. Sextants can be calibrated to find out their offset. Once you know this you can correct for the error of the sextant when you do your calculations.



What are the Other Errors?

Besides the sextant offset, there are many other sources of error. We will look at the 2 of them.

1. If you look at the picture above, you can see that the person that looks at the horizon isn't actually looking straight, but down a bit. This is because of the earth is round, not flat. The angle that you look down depends on how tall you are. If you were on top of a building that was in the middle

of a big field, you would have to look down quite a bit to see the horizon. If you lying on your stomach in the field, you wouldn't have to look down at all. You can see this by looking at the [Sextant Corrections Worksheet](#)). It's easier to use a sextant when you're standing, so the angle that you're measuring is actually larger than the true altitude. This error is called the "dip of the sea horizon." Luckily, it is really easy to figure out using the formula below:

$$DIP = 1.753\sqrt{H}$$

H is the height of your eye in meters and DIP is the correction in minutes of arc. Subtract this from the angle you measure off the sextant.

- Another source of error is the refraction effect of the atmosphere. The atmosphere of the earth bends the light coming from the sun. The sun might be below the horizon, but the atmosphere will bend the sunlight so that you still see it. Just like the DIP, this makes the altitude seem larger than it is. The amount of bending depends on the atmospheric pressure, the temperature and your altitude. A good approximation for this error is:

$$\Delta = \frac{0.267 \frac{P}{T}}{\text{TAN} \left(\text{Alt} + \frac{4.848 \times 10^{-2}}{\text{TAN}(\text{Alt}) + 0.028} \right)}$$

The triangle is called delta and is in minutes of arc
 P is the atmospheric pressure in millibars (1 atm = 1013 millibars)
 T is the temperature in degrees Kelvin
 Alt is the altitude in degrees (reading from the sextant corrected for dip.)

B. Before the activity:

- Print out Worksheets for each group or individual.
- The Excel files are protected from changes (with the exception of the data entry boxes), but the protection can be removed. If students are fairly computer savvy, a password may be added to guard the protection. If you are going to do this, do it and save *before* loading onto other computers.
 - On the **Tools** menu, point to **Protection**, and then click **Unprotect Sheet**.
 - On the **Tools** menu, point to **Protection**, and then click **Protect Sheet**.

- c. When prompted, leave all boxes checked and enter the protection password for the worksheet. Passwords are case sensitive. To unprotect the sheet again, you must type the password exactly as it was created, including uppercase and lowercase letters.
3. Load the Sextant Corrections Excel File onto all computers and put it in an easy to access place (Desktop, for instance), or better yet, have it opened up when students arrive.

Note: In the “Refraction of the Atmosphere” section, the Temperature and Pressure data boxes are not protected. This is to allow the option of investigating these variables but they are not highlighted to keep the basic lesson more focused. See [Suggestions to Scale Activity for Grades 6 to 8](#).

C. With the Students:

Before students go to the computers:

1. Divide class into groups (depending on the number of computers available) and give each student or group the [Sextant Corrections Worksheet](#).
2. Discuss the concepts of the “Dip of the Sea” correction. If students have calculators have them check the 2-meter height example answer.
3. Discuss the concepts of the “Refraction of the Atmosphere” correction. Reassure them they will NOT have to do this calculation by hand! Emphasize that the computer will be doing that calculation every time they put in a new number. This allows students to try many angles and look for trends in the results.

At the Computers:

4. Have students do the “Dip of the Sea” correction and answer the questions on the worksheet.
5. Have students do the “Refraction of the Atmosphere” correction and answer the questions at the bottom of the Worksheet.
6. Each group or individual should turn in the worksheet when complete. No print out is needed.

(For ambitious students: try doing the Refraction example equation. The 10 degrees Celsius temperature must be converted to Kelvin (283.15 degrees) when used in the equation, and all other values are as shown. Note the results are given in minutes of arc, and there are 60 minutes of arc in 1 degree.)

Math Skills Reinforced —

6th, 7th, 8th: Students will use basic trigonometric functions to determine angle measurements. In addition, they will analyze functional relationships and examine how a change in one variable results to change in another.

Activity Desired Student Outcomes —

After this activity, students should know that computers are much faster at doing math calculations than humans. When higher accuracy is needed, calculations become more difficult so using computers is even more practical. Computers allow you to try many numbers in an equation quickly — giving the operator a better chance to understand what the equation represents.

Activity Assessment & Evaluation —

Pre-Activity Assessment

- Discussion:
 - Who would believe me if I told you that when you are looking at a sunset, the sun has actually already set? (Answer: Encourage discussion: How much can we trust our eyes? Is the sun setting or are we? How could the sun have already set if we can still see it?) It is a true statement because the Earth's atmosphere refracts (bends) the rays of sunlight over the horizon so we still see them for a while after the sun has geometrically set!

Activity Embedded Assessment

- Worksheet/Computer Calculations: Students follow and complete the Excel file and worksheet.

Post-Activity Assessment

- Questions/Answers:
 - Would someone using a sextant on the moon need to make these same corrections? Why or why not? How might they be different? (Answer: The horizon dip effect would still need to be corrected on the moon and it would be larger because the moon is smaller than Earth so its horizon “dips” away even faster than Earth's. Imagine standing on a basketball! Looking down you can see almost 90 degrees around the horizon of the ball. This is a HUGE dip error. The refraction of the atmosphere correction would not be needed since the moon has almost no atmosphere.)

Suggestions to Scale Activity for Grades 6 to 8 —

- 6th Grade: Have the students do one worksheet per group and help each other.
- 7th Grade: Do activity as is.
- 8th Grade: Let students try varying the temperature and atmosphere values for the refraction correction. A normal range for these is -15 deg C to 40 deg C, and 970 mbar to 1030 mbar. Taking data and plotting these is a good way to see which affect the equation the most. Challenge students to determine why these values increase or decrease the refraction. A: Cold air is denser than hot air and high-pressure air is denser than low-pressure air; therefore, a denser atmosphere creates more refraction.

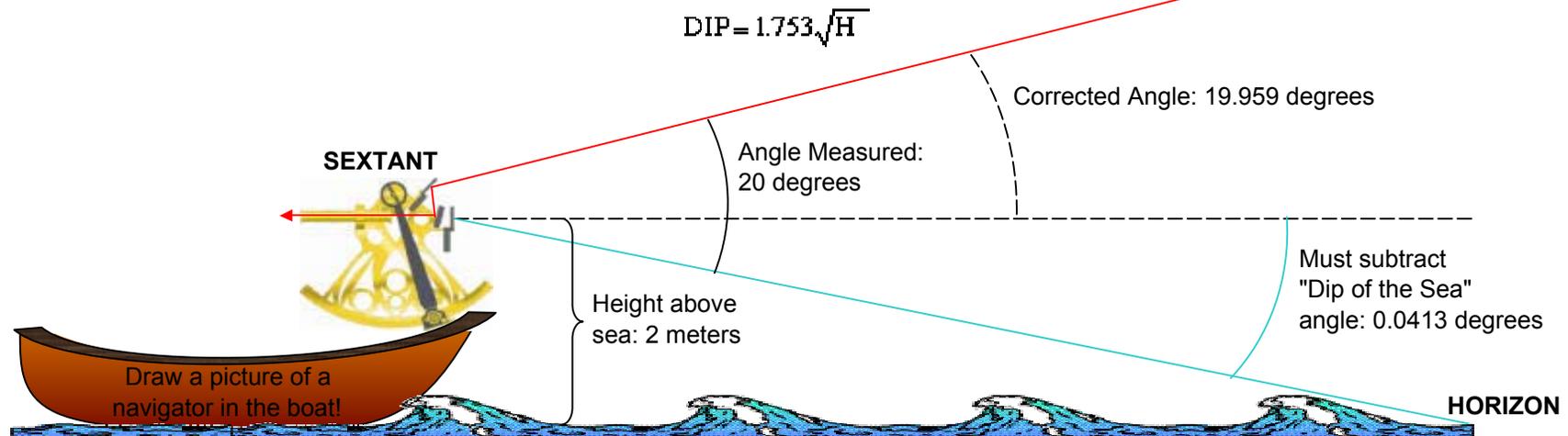
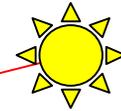
Sextant Corrections Worksheet

Name : _____

Date: _____

"Dip of the Sea Horizon"

Angle measured from horizon (degrees)	Height (H) of eyes above sea or ground (in meters)	"Dip of the Sea" in arc minutes	Divide by 60 to convert to degrees	Corrected Measurement
20	2	2.4791	0.0413	19.9587



AT THE COMPUTER: Input these heights and record the "Dip" Correction to see how increasing height affects a sextant measurement!

Height above Sea	"Dip" Correction
2 m (in the row boat)	
5 m (swabbing the deck)	
10 m (Captain's helm)	
50 m (up the mast)	
100 m (shot out of a cannon)	
300 m (hot air balloon)	
1000 m (you're in an airplane!)	

Does the "Dip" correction get bigger or smaller as height is increased? _____

At what height does the corrected measurement become a negative value? _____

If the sun had set and was 10 degrees below the horizon (so the corrected angle would be -10 degrees!) - how high (in meters) would you have to fly to see it rise to a 20 degree angle? _____ You just made the sun rise (and are well into orbit)!

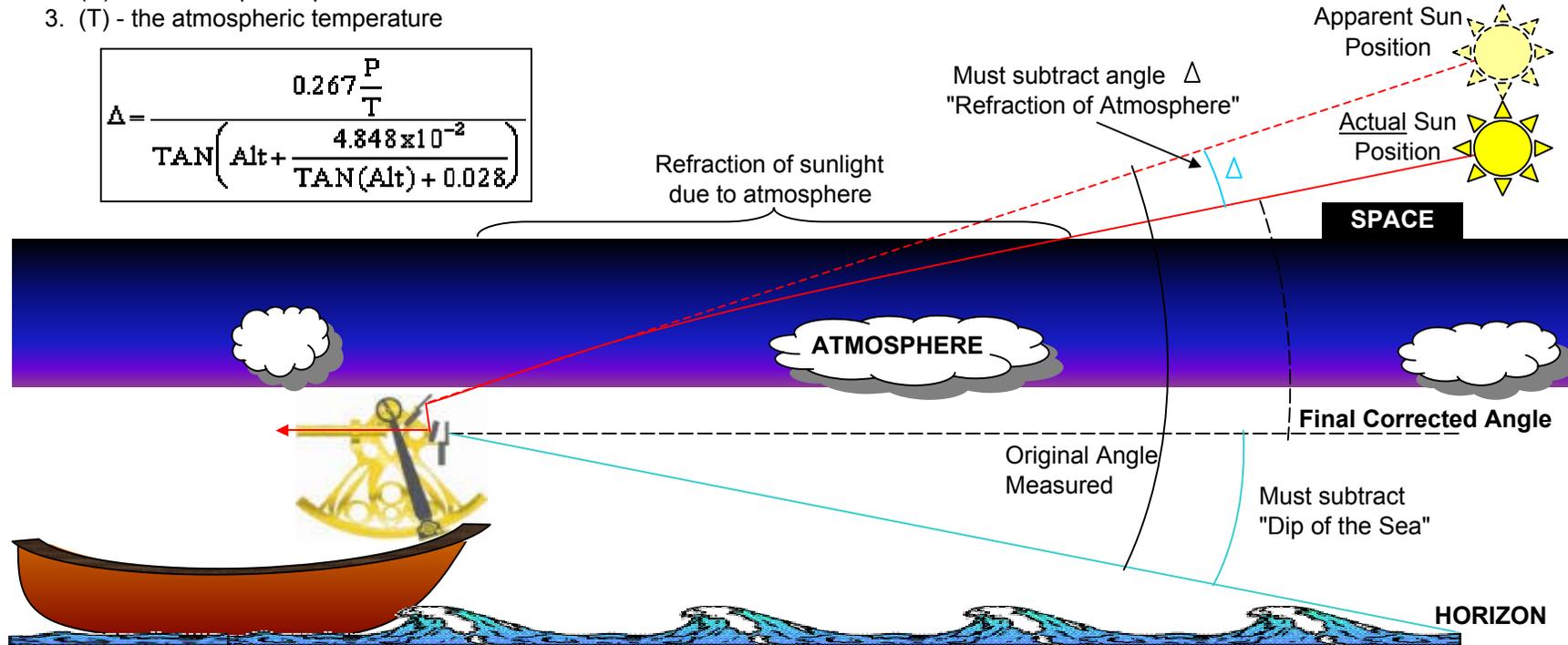
Sextant Corrections Worksheet

"Refraction of the Atmosphere"

For this correction, the math is a bit more complicated and there are 3 inputs to the equation:

1. (Alt) - the angle from the sextant - corrected for horizon dip!
2. (P) - the atmospheric pressure
3. (T) - the atmospheric temperature

$$\Delta = \frac{0.267 \frac{P}{T}}{\text{TAN} \left(\text{Alt} + \frac{4.848 \times 10^{-2}}{\text{TAN}(\text{Alt}) + 0.028} \right)}$$



Example Calculation - without these corrections, their final position would be off by 5 miles (1 arcsec = 1 mile)

Normal Sea-Level Atmospheric Pressure (millibars)	Temperature - Celsius (equivalent to 50° Fahrenheit or 283.15° Kelvin)	Original Angle Measured (degrees)	"Dip" corrected angle from before (degrees)	Refraction of atmosphere correction (arc minutes)	Correction in degrees	Final Corrected Angle (degrees)
1010	10	20	19.959	2.605	0.043	19.916

AT THE COMPUTER: Change the value in the outlined box. What happens to the correction if the sun is close to the horizon? _____

What happens if the sun is high in the sky? _____ Why is the correction larger at lower sun angles? _____

At what angle does the sun have a correction of about 1.0 arc minute (1 mile)? _____